

Ghost Equations and Diffeomorphism Invariant Theories

Olivier Piguet¹

*Universidade Federal do Espírito Santo (UFES) ,
CCE, Departamento de Física,
Campus Universitário de Goiabeiras - BR-29060-900 - Vitória - ES - Brasil.*

E-mail: `piguet@cce.ufes.br`

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Abstract Four-dimensional Einstein gravity in the Palatini first order formalism is shown to possess a vector supersymmetry of the same type as found in the topological theories for Yang-Mills fields. A peculiar feature of the gravitational theory, characterized by diffeomorphism invariance, is a direct link of vector supersymmetry with the field equation of motion for the Faddeev-Popov ghost of diffeomorphisms.

1 Introduction

In the first order formalism of four-dimensional gravitation theory [1], the independent dynamical variables are the vierbein 1-form E (giving the metric G) and the connection 1-form A . The vierbein dependence of the connection is given by the field equation with respect to A , whereas Einstein equation results from the field equation with respect to E . The action will be written, following [2, 3], in a “topological” form, i.e. in such a way that it can be interpreted as an action of the 1-form fields E and A on a differentiable manifold M , without reference to any a-priori background metric. The latter point is known [4] to be an essential characteristic of topological theories, and trying to exploit this feature belongs to the spirit of the modern attempts towards a construction of quantum gravity (see [5, 6] for reviews and further references).

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²Sign in eq. (3.9) corrected; references added.

Since the theory possesses two local symmetries – the diffeomorphism and local Lorentz invariances – one has to perform a gauge fixing for both. We will choose a gauge fixing of the Landau type, within the BRST formalism [7, 8]. Much in the same way as in topological theories, this requires the introduction of a nondynamical, background metric g . This construction closely parallels the one performed for the Chern-Simons theory in [9]. It should be clear that the background metric, being introduced only in the gauge fixing part of the theory, should not affect in any way the physical outcome, as it has been shown for instance in [9] for the – perturbative – quantum version of the Chern-Simons theory.

An interesting features of topological theories such as Chern-Simons or BF theory, is the presence of a “vector supersymmetry” – a supersymmetry whose generator is a vector valued operator [10]. In case the manifold admits isometries generated by Killing vectors – e.g. the space-time translations if the background metric is flat – the vector supersymmetry is a symmetry of the gauge-fixed action. It happens that its generator together with the BRST symmetry generator form an algebra which closes on the generators of the isometries of the translation type [9]. The vector supersymmetry has been shown to play a key role in the ultraviolet finiteness of the topological theories [11, 9].

Another interesting features – actually shared by any gauge theory, provided its gauge fixing be of the Landau type – is the so-called ghost equation [12, 8], which restricts the coupling of the ghosts and implies the nonrenormalization of their field amplitude³.

The purpose of the present note is to show the existence of such a vector supersymmetry for Einstein gravity in the Palatini formalism. We shall in fact see that the vector supersymmetry is a direct consequence of the field equation of the Feynman-Dewitt-Faddeev-Popov ghost [13] associated to diffeomorphism invariance. On the other hand, the ghost equation related to local Lorentz invariance will be seen to be algebraically associated with rigid Lorentz invariance. We shall also see that this supersymmetry, like in topological theories, yields the Sorella operator [14] δ used in order to solve the BRST cohomology and to construct the invariants of the theory. The operator δ has been given for gravity in [15, 16].

To the contrary of the topological theories of a Yang-Mills connection (Chern-Simons or BF), where the supersymmetry generators are the components of a vector and where the superalgebra closes on the translations, Einstein gravity in the Palatini formalism studied in the present paper will be seen to admit a supersymmetry possessing generators which are components of one vector and one antisymmetric tensor, the full algebra containing now all the ten Poincaré generators – in the case of a flat background metric at least⁴.

³A review of the properties of topological theories mentionned above may be found in Chapters 6 and 7 of the book [8].

⁴Such a tensor supersymmetry has been pointed out in [17], in the case of a Chern Simons

Although the present work will only be concerned with the classical aspects of the theory, the results are of interest since, as we already said, they reveal the link between the construction of the observables via the δ operator of Sorella [14, 15, 16], on one hand, and the gauge fixing, through the ghost equation, on the other hand.

2 Symmetries, Gauge Fixing and BRST Invariance

The Einstein gravity Lagrangian in the first order formalism of Palatini [1] may be written as [2, 3]:

$$S_{\text{inv}} = \frac{1}{4} \int_M \varepsilon_{IJKL} E^I \wedge E^J \wedge F^{KL}(A) + S_{\text{matter}}(E, A, \Phi) . \quad (2.1)$$

The integral is taken over some differentiable 4-manifold M , E^I is the vierbein 1-form, with $I = 0, \dots, 3$ a tangent plane Lorentz index. F^{KL} is the curvature 2-form

$$F^{IJ}(A) = dA^{IJ} + A^{IK} \wedge A_K^J \quad (2.2)$$

of the Lorentz connection⁵ A^{IJ} , the latter being taken as an independant variable⁶. ε_{IJKL} is the rank four totally antisymmetric tensor, normalized by $\varepsilon_{0123} = 1$. In the following, the exterior multiplication symbol \wedge will be omitted. S_{matter} is some action for minimally coupled matter fields Φ , which we don't need to specify. We shall in fact omit this part in the following, for the sake of simplicity.

The field equations given by the variations of this action read

$$\begin{aligned} \frac{\delta S_{\text{inv}}}{\delta E^I} &= \frac{1}{2} \varepsilon_{IJKL} E^J F^{KL} , \\ \frac{\delta S_{\text{inv}}}{\delta A^{IJ}} &= \varepsilon_{IJKL} E^K D E^L , \end{aligned} \quad (2.3)$$

where D is the covariant exterior derivative: $D E^I = d E^I - A^I_J E^J$. It is known [1, 3] that they lead to the usual specification of a torsion-free connection function of the vierbein⁷ and to the Einstein equation, in a Riemanian space-time with metric

$$G_{\mu\nu} = E_\mu^I E_\nu^J \eta_{IJ} , \quad (2.4)$$

model in a gravitational background in the vielbein formalism.

⁵If the connection is self-dual, (2.1) is the Ashtekar action [18, 2].

⁶In a particular coordinate frame with $x = (x^\mu, \mu = 0, \dots, 3)$, $E^I = E_\mu^I dx^\mu$, $F^{IJ} = \frac{1}{2} F_{\mu\nu}^{IJ} dx^\mu \wedge dx^\nu$, etc.

⁷This is true for pure gravity. In case of coupling with matter, the second field equation does not automatically lead to a vanishing torsion [1]. One may then choose to stay with a non-vanishing action, or to impose a supplementary condition.

where η_{IJ} is the Minkowsky metric⁸ $\text{diag}(1, -1, -1, -1)$, used to lower and rise the tangent space indices I, J, \dots .

The action (2.1) is invariant under the diffeomorphisms, written in infinitesimal form, the infinitesimal parameter being a vector field ξ :

$$\delta_{(\xi)}\varphi = \mathcal{L}_\xi\varphi, \quad \varphi = E^I, A^{IJ}, \quad (2.5)$$

where \mathcal{L}_ξ is the Lie derivative along the vector ξ . It is also invariant under the local Lorentz transformations – written in infinitesimal form, with local parameters $\omega^{IJ} = -\omega^{JI}$:

$$\begin{aligned} \delta_{(\omega)}E^I &= \omega^I{}_J E^J, \\ \delta_{(\omega)}A^{IJ} &= d\omega^{IJ} + \omega^I{}_K A^{KJ} + \omega^J{}_K A^{IK}. \end{aligned} \quad (2.6)$$

In view of the gauge fixing procedure it is convenient to express these local invariances in the form of a nilpotent BRST operation s defined by [19]:

$$\begin{aligned} sE^I &= \mathcal{L}_\xi E^I + \omega^I{}_J E^J, \\ sA^{IJ} &= \mathcal{L}_\xi A^{IJ} + d\omega^{IJ} + \omega^I{}_K A^{KJ} + \omega^J{}_K A^{IK}, \\ s\xi &= \frac{1}{2}\{\xi, \xi\}, \quad (\text{or: } s\xi^\mu = \xi^\lambda \partial_\lambda \xi^\mu), \\ s\omega^I{}_J &= \mathcal{L}_\xi \omega^I{}_J + \omega^I{}_K \omega^K{}_J, \end{aligned} \quad (2.7)$$

with $s^2 = 0$. The infinitesimal parameters $\xi^\mu(x)$ – the components of the vector ξ – and $\omega^I{}_J(x)$ are now Grassmann (i.e. anticommuting) number fields – the Faddeev-Popov ghosts. The bracket $\{, \}$ is the Lie bracket⁹.

In order to gauge fix the theory with respect to its local symmetries – diffeomorphism and local Lorentz invariances – we introduce antighosts¹⁰ $\bar{\xi}_I$, $\bar{\omega}_{IJ}$ and Lagrange multipliers λ_I , b_{IJ} , with the following nilpotent BRST transformations:

$$s\bar{\xi}_I = \lambda_I, \quad s\lambda_I = 0, \quad s\bar{\omega}_{IJ} = b_{IJ}, \quad sb_{IJ} = 0. \quad (2.8)$$

The gauge fixing part of the action is then defined as:

$$\begin{aligned} S_{gf} &= -s \int_M d^4x \sqrt{-g} g^{\mu\nu} \left(\partial_\mu \bar{\xi}_I E_\nu^I + \frac{1}{2} \partial_\mu \bar{\omega}_{IJ} A_\nu^{IJ} \right) \\ &= - \int_M d^4x \sqrt{-g} g^{\mu\nu} \left(\partial_\mu \lambda_I E_\nu^I + \frac{1}{2} \partial_\mu b_{IJ} A_\nu^{IJ} \right) \\ &\quad + \int_M d^4x \sqrt{-g} g^{\mu\nu} \left(\partial_\mu \bar{\xi}_I sE_\nu^I + \frac{1}{2} \partial_\mu \bar{\omega}_{IJ} sA_\nu^{IJ} \right), \end{aligned} \quad (2.9)$$

⁸We consider a Lorentzian signature. But everything applies as well to the Euclidean case.

⁹In a particular coordinate frame, the Lie bracket of 2 vectors u, v takes the form

$$\{u, v\}^\mu = u^\lambda \partial_\lambda v^\mu \pm v^\lambda \partial_\lambda u^\mu,$$

with the sign $+$ is both u e v are odd, and the sign $-$ otherwise. Even (odd) refers to the commuting (anticommuting) character of the object.

¹⁰Despite of an unfortunate but usual terminology, the antighosts are independent of the ghosts [20].

which is automatically BRST invariant. Note that in order to contract the world indices μ, ν we have introduced a (BRST-invariant) background metric $g_{\mu\nu}$ – not to be confounded with the physical, dynamical metric $G_{\mu\nu}$ defined in (2.4).

This particular gauge fixing, which is of the Landau type, is completely determined by BRST invariance and by the “gauge conditions” – i.e. by the field equations for the Lagrange multipliers:

$$\frac{\delta S}{\delta \lambda_I} = \partial_\mu \left(\sqrt{-g} g^{\mu\nu} E_\nu^I \right) , \quad \frac{\delta S}{\delta b_{IJ}} = \partial_\mu \left(\sqrt{-g} g^{\mu\nu} A_\nu^{IJ} \right) , \quad (2.10)$$

where S is the total action

$$S = S_{\text{inv}} + S_{\text{gf}} , \quad (2.11)$$

which is BRST invariant by construction: $sS = 0$.

On the other hand, the field equations for the ghosts ξ and ω are

$$\begin{aligned} \frac{\delta S}{\delta \xi^\mu} &= \left(-\sqrt{-g} g^{\nu\lambda} \partial_\lambda \bar{\xi}_I \partial_\mu E_\nu^I + \partial_\nu \left(\sqrt{-g} g^{\nu\lambda} \partial_\lambda \bar{\xi}_I E_\mu^I \right) \right) \\ &\quad - \frac{1}{2} \sqrt{-g} g^{\nu\lambda} \partial_\lambda \bar{\omega}_{IJ} \partial_\mu A_\nu^{IJ} + \frac{1}{2} \partial_\nu \left(\sqrt{-g} g^{\nu\lambda} \partial_\lambda \bar{\omega}_{IJ} A_\mu^{IJ} \right) , \\ \frac{\delta S}{\delta \omega^{IJ}} &= -\sqrt{-g} g^{\mu\nu} \left(\partial_\mu \bar{\xi}_I E_{\nu J} + \partial_\mu \bar{\omega}_{IK} A_{\nu J}^K - (I \leftrightarrow J) \right) + \partial_\mu \left(\sqrt{-g} g^{\mu\nu} \partial_\nu \bar{\omega}_{IJ} \right) . \end{aligned} \quad (2.12)$$

3 Ghost Equation and Vector Supersymmetry

Let us introduce the condensed notation

$$\begin{aligned} \{\mathcal{A}_\mu^i, i = 1, 2\} &= \{E_\mu^I, A_\mu^{IJ}\} , \\ \{\bar{C}_i, i = 1, 2\} &= \{\bar{\xi}_I, \bar{\omega}_{IJ}\} , \quad \{B_i, i = 1, 2\} = \{\lambda_I, b_{IJ}\} , \end{aligned} \quad (3.1)$$

under which the gauge conditions (2.10) and the equation for the ghost ξ (2.12) now read¹¹

$$\frac{\delta S}{\delta B_i} = \partial_\mu \left(\sqrt{-g} g^{\mu\nu} \mathcal{A}_\nu^i \right) , \quad (3.2)$$

$$\frac{\delta S}{\delta \xi^\mu} = \sum_{i=1,2} \left(-\sqrt{-g} g^{\nu\lambda} \partial_\lambda \bar{C}_i \partial_\mu \mathcal{A}_\nu^i + \partial_\nu \left(\sqrt{-g} g^{\nu\lambda} \partial_\lambda \bar{C}_i \mathcal{A}_\mu^i \right) \right) . \quad (3.3)$$

What we claim here is that, under circumstances to be specified later on, the theory is invariant under the *vector supersymmetry transformations* (we use the notation

¹¹Summations \sum_I and $\frac{1}{2} \sum_{IJ}$ over repeated indices are implicit.

(3.1))

$$\begin{aligned}\delta_{(\varepsilon)}^S \xi^\mu &= \varepsilon^\mu , \\ \delta_{(\varepsilon)}^S B_i &= \varepsilon^\mu \partial_\mu \bar{C}_i , \\ \delta_{(\varepsilon)}^S \varphi &= 0 , \quad \varphi \neq \xi^\mu , B^i ,\end{aligned}\tag{3.4}$$

where the infinitesimal parameter ε^μ is a vector field – taken as commuting, to the contrary of ξ^μ , so that the supersymmetry operator $\delta_{(\varepsilon)}^S$ is an antiderivation. The latter together with the BRST operator s obey the superalgebra anticommutation relations

$$s^2 \varphi = 0 , \quad (\delta_{(\varepsilon)}^S)^2 \varphi = 0 , \quad \{s, \delta_{(\varepsilon)}^S\} \varphi = \mathcal{L}_\varepsilon \varphi ,\tag{3.5}$$

for all fields φ , where \mathcal{L}_ε is the Lie derivative along the vector ε .

In order to check the possible invariance of the action under the vector supersymmetry, we first note that this is trivially the case for the gauge invariant part (2.1) of the total action (2.11). Next, since the gauge fixing part (2.9) is a BRST variation:

$$S_{\text{gf}} = - \int_M d^4x \sqrt{-g} g^{\mu\nu} \sum_{i=1,2} s \left(\partial_\mu \bar{C}_i \mathcal{A}_\nu^i \right) ,\tag{3.6}$$

we can use the anticommutation relation (3.5) and thus write

$$\delta_{(\varepsilon)}^S S = - \int_M d^4x \sqrt{-g} g^{\mu\nu} \sum_{i=1,2} \mathcal{L}_\varepsilon \left(\partial_\mu \bar{C}_i \mathcal{A}_\nu^i \right) .\tag{3.7}$$

Partial integrations¹² then yield

$$\delta_{(\varepsilon)}^S S = \int_M d^4x \sqrt{-g} \left(\mathcal{L}_\varepsilon g^{\mu\nu} + \nabla_\lambda^{(g)} \varepsilon^\lambda g^{\mu\nu} \right) \sum_{i=1,2} \left(\partial_\mu \bar{C}_i \mathcal{A}_\nu^i \right) ,\tag{3.8}$$

where $\nabla_\mu^{(g)}$ is the covariant derivative with respect to the background metric $g_{\mu\nu}$. The first conclusion is that, generically, the vector supersymmetry transformations (3.4) are not an invariance of the theory. However they will indeed represent an invariance if and only if the parenthesis in the integrant of the right-hand side of (3.8) vanishes:

$$\mathcal{L}_\varepsilon g^{\mu\nu} + \nabla_\lambda^{(g)} \varepsilon^\lambda g^{\mu\nu} = 0 ,\tag{3.9}$$

which is easily shown to be equivalent to the condition that the vector ε be a Killing vector field of the background metric: $g^{\mu\nu}$:

$$\mathcal{L}_\varepsilon g^{\mu\nu} = 0 , \quad \text{or:} \quad \nabla_\mu^{(g)} \varepsilon_\nu + \nabla_\nu^{(g)} \varepsilon_\mu = 0 .\tag{3.10}$$

In such a case, the vector supersymmetry invariance may be expressed by the functional identity (still using the notation (3.1))

$$\delta_{(\varepsilon)}^S S \equiv \int_M d^4x \varepsilon^\mu \left(\frac{\delta}{\delta \xi^\mu} + \sum_{i=1,2} \partial_\mu \bar{C}_i \frac{\delta}{\delta B_i} \right) S = 0 .\tag{3.11}$$

¹²We assume allthrough the absence of boundary terms contributions.

It is illustrative to consider a flat background metric, e.g. the Minkowski one: $g_{\mu\nu} = \eta_{\mu\nu}$. In this case the general solution of the condition (3.10) reads

$$\varepsilon^\mu = a^\mu + b^{\mu\nu} x_\nu, \quad \text{with} \quad a^\mu, b^{\mu\nu} = -b^{\nu\mu} \quad \text{constants.} \quad (3.12)$$

The right hand side of the anticommutator in (3.5) is then an infinitesimal rigid Poincaré transformation of parameters a^μ and $b^{\mu\nu}$.

Note that one could have derived the identity (3.11) directly from the equation (3.3) for the diffeomorphism ghost ξ , integrated with the vector field ε , and performing some partial integrations. Thus we clearly see how the vector supersymmetry is linked to the diffeomorphism ghost equation. In this respect the situation differs from the one encountered in Yang-Mills topological theories (Chern-Simons, BF), where there is no such narrow relation between a ghost equation and the vector supersymmetry – although they both hold in a Landau type gauge, too.

Another difference with the topological Yang-Mills case is that the weaker condition (3.10) for supersymmetry invariance holds in the present case, whereas it reads there $\nabla_\mu^{(g)} \varepsilon_\nu = 0$: the superalgebra (3.5) closes there on isometries of the translation type only - true translations instead of general Poincaré transformations for a flat background metric.

It is known that in the Yang-Mills case the vector symmetry operator may be expressed in the form of the so-called operator δ of Sorella [14] used to construct the invariants of the theory, and characterized by the algebraic relation

$$[\delta, s] = d, \quad (3.13)$$

where d is the exterior derivative. In the present case, too, there exists [15, 16] such an operator δ . And, remarkably, it is linked to our vector supersymmetry, hence to the diffeomorphism ghost equation, in the following way. Considering the supersymmetry transformation rules (3.4) for a constant¹³ vector field ε , we define the action of the operator δ as given by these transformations, with ε^μ replaced by the differential dx^μ :

$$\begin{aligned} \delta \xi^\mu &= dx^\mu, \\ \delta B_i &= d\bar{C}_i, \\ \delta \varphi &= 0, \quad \varphi \neq \xi^\mu, B^i, \end{aligned} \quad (3.14)$$

which is the result of [15, 16] – up to the action on the Lagrange multiplier fields B_i , not considered there. We can easily check the commutation rule (3.13), and also that δ commutes with d . Note that, as in [16], we can write the first of eqs. (3.14) in a coordinate independent way as

$$\delta \eta^I = E^I, \quad (3.15)$$

¹³Vector supersymmetry invariance will then hold for a flat Minkovskian background metric.

where $\eta^I \equiv E_\mu^I \xi^\mu$ is the “tangent space translation ghost” [19, 16].

Before concluding, we could ask for the role of the Lorentz ghost equation, the second of eqs. (2.12). The answer is much the same as in ordinary gauge theories for the Yang-Mills ghost equation [12]. Integrating the Lorentz ghost equation in space-time, integrating by part and using the gauge conditions (2.10), we obtain

$$\delta_{IJ}^{(\text{SL})} S \equiv \int_M d^4x \left(\frac{\delta}{\delta \omega^{IJ}} - \bar{\xi}_I \frac{\delta}{\delta \lambda^J} + \bar{\xi}_J \frac{\delta}{\delta \lambda^I} - \bar{\omega}_{IK} \frac{\delta}{\delta b^K_J} + \bar{\omega}_{JK} \frac{\delta}{\delta b^K_I} \right) S = 0, \quad (3.16)$$

which is very similar to the result of [12], the Yang-Mills gauge invariance being now replaced by the local Lorentz invariance. One also may check the anticommutation rule

$$\{s, \delta_{IJ}^{(\text{SL})}\} = \delta_{IJ}^{(\text{L})}, \quad (3.17)$$

where the right-hand side is an infinitesimal generator of rigid Lorentz transformation.

4 Conclusions

We have found a direct relation between the diffeomorphism ghost equation (the first of eqs. (2.12)) and the existence of a vector supersymmetry (3.4) – or of the Sorella operator δ (3.14), (3.15). This appears to be a characteristic features of theories invariant under “active diffeomorphisms”, i.e. diffeomorphisms which act on the dynamical fields only¹⁴. In such theories, the diffeomorphism ghost ξ is a dynamical field, which actually means that it obeys an equation of motion.

Our results do not depend on the specificity of the invariant action taken to define the theory. They obviously also hold in the case of a self-dual connection A^{IJ} , in which case the action (2.1) is that of Ashtekar [18, 1, 2]. As we have mentioned, the presence of minimally coupled matter is allowed, as well as other type of actions, provided they share the same “topological-like” character, i.e. provided they are constructed with the vierbein and connection as independent variables, and with invariance under active diffeomorphisms,

We have also emphasized the differences of the present diffeomorphism invariant theory with respect to the topological theories for Yang-Mills fields, such as the role of the ghost equation, but also the existence of more supersymmetry thanks to a weaker condition of invariance.

During completion of this work, the author became aware of a recent preprint [21] – published by now – which gives an alternative derivation of vector supersymmetry

¹⁴In a quantum context, these are diffeomorphisms acting quantum mechanically on the field operators – vierbein, connection and matter fields [5].

in topological theories. This method may be applied to the gravitational case, too – and has been indeed applied in the published version of [21].

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